THE MATHEMATICS TEACHER

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EDITORIAL.

Investigations.

In this number of the magazine there appears a preliminary report of the Committee on Testing the Results of Geometry Teaching. This committee has undertaken one of the most difficult investigations under way in this country. Scientific investigation of the results obtained from the study of various subjects is a comparatively new branch of education, and if its conclusions prove to be dependable it is likely to revolutionize curricula and methods of teaching.

The *Mathematics Teacher* bespeaks the support of all its readers in behalf of committees like this one, which can succeed only through the co-operation of large numbers of teachers who will give their time and their thought to the problem under investigation.

PRELIMINARY REPORT OF THE COMMITTEE ON TESTING THE RESULTS OF GEOMETRY TEACHING.

This committe was appointed for the purpose of testing in as scientific a manner as possible the results obtained from the study of geometry. The right of geometry to the prominent place which it has hitherto held in the curricula of most high schools and preparatory schools has been challenged on the ground that it does not provide for the ordinary student an essential element of his education which could not be provided more satisfactorily in some other manner. Also the traditional method of teaching geometry has been adversely criticized and several new methods have been suggested and to some extent put into practice. This committee has assumed that its work consisted primarily in testing the results of geometry teaching in order to determine whether those criticisms are justified. To reach reliable conclusions in regard to the position of geometry in the curricula, it is necessary that tests be devised of a non-geometrical nature so that they could be given to students whether they have studied geometry or not. To test the second point, some of the tests at least should be geometrical in character but the non-geometrical tests, given for the first purpose, would probably also throw light on the second.

The committee considered what particular results of geometry teaching were most likely results unique to the subject of geometry, and to answer that question, a number of prominent mathematics teachers were asked to state definitely their opinion on that point. After receiving a number of replies, the committee selected two such results for the purpose of testing, namely:

Ability to reason logically, and

Ability to express one's self with precision.

Under the general head of ability to reason logically, the committee would include the following statements which have been made by different individuals but which seem to be inseparable as far as testing purposes are concerned, namely:

Ability to tell whether a given argument is logically sound.

Ability to do logically sound thinking.

Ability to construct a train of reasoning from given material.

One of the first tasks, therefore, which the committee has undertaken is that of preparing suitable non-geometrical tests to determine the results as to reasoning ability, and considerable difficulty has been experienced in preparing such tests. Indeed the committee is not sure that it is possible to prepare reliable tests of that character.

The following five questions indicate the nature of such tests as far as the committee has been able to devise them or find them already prepared:

Non-Geometrical Questions for Testing Reasoning Ability.

1. If John agrees to join the football team provided Charles joins it, but Charles decides not to join it, what follows about John? If John joins, but Charles does not join, is John breaking his agreement? (Modified from the first preliminary report of this committee.)

2. If all solids were crystals, then which of the following statements would necessarily be true, which could not be true, and which might be true:

(1) All crystals are solids.
(2) Some solids are not crystals.
(3) Some things are neither crystals nor solids.
(4) No crystals are liquids.
(5) Some solids are crystals.

3. All the members of a club are invited to compete in a certain athletic contest. Some members of the club are women. What conclusions (if any) can be drawn from the above statements?

4. Do you discover any defects in the reasoning in the following? Bullies are always cowards, but not always liars; therefore, liars are not always cowards.

(Numbers 2, 3, and 4 are modifications of suggested tests given in a paper by Professor Metzler in *The Journal of Educational Psychology.*)

5. A certain club wishes to select the evening of the week for

its regular meeting which will be most satisfactory to its membership. The secretary wrote to each member, asking that each member let him know what evening would be most satisfactory. Can you suggest any other information that would have been better for the secretary to have asked for, and if so, why?

The committee desires very greatly that each member of this Association send to it questions of a non-geometrical character which could be given to high school students. When a large collection of such questions has been gotten together by the committee, an effort will then be made to test each of these questions in regard to its reliability in indicating the reasoning power of persons to whom the question might be given. The committee has in mind, when speaking of testing these questions, the carrying out of some correlation tests such as those explained in the recent books on mental measurements.

After each question has been tested and the coefficient of reliability for each determined, then the committee will group those questions whose coefficient of reliability is sufficiently high, into sets such that a whole set of them might be given to a class in a reasonable time, say one hour, and such that the set would probably give, in the case of each student, a reliable estimate of that student's reasoning ability. These sets of questions would then be given to several hundred persons, indeed possibly to several thousand persons, if sufficient assistance will be rendered the committee by the members of this Association who are teaching high school pupils.

These tests should of course be given under very definite and exactly known conditions and the committee has prepared a set of rules under which the tests would be given and the papers marked. A copy of these rules accompanies this report and the committee would be very much pleased to have suggestions from the Association that would help to improve them.

The committee has also planned to obtain in the case of each individual who takes these tests certain information which would assist in analyzing the results when they are obtained. Statements of the information to be asked for accompany this report.

If these tests should show that the pupils who had studied geometry were superior in reasoning ability, the question would still not be fully settled as to whether that superiority was due

to the fact that they had studied geometry, or was due to natural ability to reason logically. With this in view, the committee has planned to give to some classes who are studying geometry a succession of tests; say, one before beginning the study of geometry, others at various times during the course, and one at the conclusion of the subject; to determine whether or not the ability to reason on non-geometrical subject matter was increased during the study of geometry.

The committee has planned to give also a test of a non-geometrical character which would aim to determine whether or not students who had studied geometry had the ability to express themselves with greater precision than those who had not studied geometry.

The plans for this test are simply that an ordinary elementary test in the writing of English composition would be given by giving to the pupils a list of subjects about any one of which they might write a few hundred words, allowing them to select one subject and then write a short composition on it. This composition would then be marked with reference to its precision of expression by persons experienced in marking composition papers, and the results tabulated according to whether the student had studied geometry or not.

To carry out these plans, a considerable period of time will be needed. Also a large amount of work and an appreciable expense will be involved. The committee has been seeking other associations, both abroad and in this country, with whom some plans of co-operation may be formed so that the work may be done more effectively and incidentally the labor and expense be distributed more widely. Moreover the committee has learned of one research student and is seeking others who may possibly carry out a large part of the investigation in connection with their researches. However, the approval of this report and the instruction of the committee to proceed will be interpreted as carrying with it an obligation on the part of the Association to meet the necessary expense.

Rules for Carrying Out the Tests.

Note: These are the rules to be followed after the coefficients of reliability for individual questions have been determined and

the questions have been formed into sets as explained in the report.

1. Each person taking the tests should be provided with examination books of uniform size and quality, which should be numbered so that every person taking a test will thereby be assigned a unique number, and neither the person's name nor any information about him should be placed on it.

2. There should be filled out, for each person taking a test, a blank form (see form 1), giving certain information about him.

3. There should also be filled blank forms (see form 2 or form 3), giving certain other information about each person taking a test. One blank will be sufficient for a whole class or any group in which the information called for is common to every individual of the group.

4. Each paper should be read by two or more experienced readers and a mark recorded by the proper identifying number. The reader should have no information about the persons whose papers he reads and no reader should place on the papers any marks or notations of any kind.

5. The marks obtained by those readings, together with the information on the accompanying forms, should then be tabulated and analyzed by expert statisticians.

Testing the Results of Geometry Teaching.

Form 1.

(The name and identifying number should be filled in by the person taking the test before or just when he hands in his paper; the remainder can be filled in at any time.)

Name of person taking the test ....

Identifying number found on the examination book used by him........

Age ....

Has he taken a course in plane geometry which consisted chiefly of the usual logical proofs? ....

Age when that course in geometry was begun ....

Any school activities that might have specially influenced his mental qualities ....

....

Any activities outside of school that might have a special bearing on his mental qualities ....

TESTING THE RESULTS OF GEOMETRY TEACHING.

Form 2.

(For persons who have studied geometry.)

(In most cases to be filled out by the teacher.)

The following information applies in common to all those persons whose identifying numbers are in the following list of numbers.........
....

1. Name the text-book used ....
2. How much time has been given to recitations in geometry?
................periods per week for................weeks
3. If a full text or a suggestive text was used, were
   (*a*) theorems assigned for study from the text before class development? ....
   (*b*) theorems developed in class before assignment for study? .....
4. Were methods of reasoning and methods of attack given specific discussion? ....
5. Were any so-called originals done
   (*a*) in class discussion? ....
   (*b*) individually? ....
6. What other subjects have been studied that might have helped appreciably in developing their reasoning ability? ....
7. If geometry was studied during the week immediately preceding the test, give on the back of this blank a specific statement of the exact work during the recitation period and in home study for that week? ....
....

TESTING THE RESULTS OF GEOMETRY TEACHING.

Form 3

(For persons who have not studied geometry.)

The following information applies in common to all those persons whose identifying numbers are in the following list of numbers.........
....

1. What subjects have been studied that might have helped appreciably in developing their reasoning ability? ....
....

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THE COMPREHENSIVE EXAMINATION IN MATHEMATICS.

By Ernest H. Koch, Jr.

My topic, like every other educational problem, is an old one in a new dress. We would make no progress in educational matters if it were not for the fact that our educational systems are undergoing readjustments in accordance with the demands of life.

Three thousand years ago China examined her candidates for public service by subjecting them to a systematic test. Since that time we have continued to examine candidates for service, promotion and honors by subjecting them to various kinds of examinations which are typefied by the adjective modifiers, superficial, brief, thorough, protracted and searching. We must add one more, the comprehensive examination, which represents the very last word in educational thought.

The elements of our examinations are traceable to distinct periods in the history of education. The method of conducting examinations in oral or written form, in practical test and by thesis or any combination of these is a matter of expediency.

A *comprehensive examination* is a rational attempt to test the degree of adjustment of an individual in the scheme of life. Such an examination should reflect the sociological and psychological aims of education, and then their purpose, necessity and utility will become comprehensive in name as well as in facts, not only to the teachers but also to the greater bodies of students and laymen. As soon as we take a more sympathetic attitude with the realities of life then we appreciate that success in life depends not only upon capacity and strength but also upon the fitness of the individual. You will agree with me then that what we should do in our examinations is not only to test for range and depth of knowledge but also to ascertain the individual's powers of co-ordination, reflection, expression, common-sense, skill in manipulation, application, and higher powers of investigation. We are meeting daily the inquiry on the efficiency of our school work by reiterating little set speeches. At the

same time we are deceiving ourselves, because we know that the majority of examinations set false standards which not only negate good teaching but retard the stimuli which nature intended should be utilized to promote effort, scholarship, character and activity in the larger problems of citizenship. The comprehensive examination is the first instrument that we have forged by means of which we hope to achieve some real success in this business of education. It suggests the means for making our training efficient and in deciding the important factors in education and their limitations. We must gain and then maintain a sympathy with the problems of life so that the transition from the school world to the larger environment may require a minimum readjustment. The power of comprehensive vision is what we cherish. The knowledge which we have imparted and the character training which has been developed is of no consequence unless there is back of it the power to discern the true relation of things. Would you expect a great electric company to install an expensive hydro-electric plant on the banks of a mighty stream and then leave it inoperative without any directing force to make a judicious distribution of the available power among the many related industries which it has promoted and made dependent upon it?

Our examinations have not been comprehensive because we have failed to take a comprehensive view of knowledge. We have been content to regard the facts of history, science, politics, religion and literature as if they were distinct entities and therefore unrelated. We have failed therefore because we have neglected to follow five fundamental considerations:

1. The child is entitled to his scientific inheritance, to his literary inheritance, to his æsthetic inheritance, to his institutional inheritance and to his religious inheritance. This body of experiences is of no value to the individual unless he can discern the general law of truth which binds and permeates them. What is this symbol of truth from which all the fabrics of knowledge are woven?

2. The true purpose of examinations is to indicate the quality and measure the quantity of individual adjustment to an arbitrary standard determined by the aim of education. This implies the elimination of conflicting aspects of education as interpreted by primary, secondary and collegiate institutions.

3. "All knowledge is either envisaged or implied in data of experiences, and it is man's reasoning faculties which make explicit what is thus implied. Man is the measure of all things."

4. The efficiency of any examination system unless wisely administered by a philosopher is subject to the frailties of human judgment. We must be mindful of the paradox which states that every individual is a triplicity of identities. He is what he seems to be to others, he is what he thinks he is and he is what he is.

5. The form and formalism of an examination should always be free from the vagaries of personal idiosyncracies. The place of examinations in the educational system, their duration, their gradation and their flexibility should depend upon the exigencies of the occasion. The complete or partial substitution of a certificate or its equivalent is a recognition of the efficiency of the institution's internal periodic examinations.

These remarks may suffice to introduce us to the more special topic of the comprehensive examination in mathematics. I hope my desire to be brief has not made my introduction seem fragmentary nor so comprehensive as to be extralimitary to our experiences.

Is mathematics justified in our curriculum and if so what are its sociological and psychological values? The answer to this question must state: (1) the meaning of mathematics; (2) the aims of mathematics teaching; (3) the relation of mathematics to other departments of knowledge; (4) the meaning of a comprehensive examination in mathematics; (5) the reaction of rational mathematics upon the comprehensive examination.

The educational value of mathematics is both utilitarian and cultural. The metrical characteristics of mathematics contribute largely to the sociological aspects, whereas the non-metrical characteristics of mathematics contribute largely in the psychological aspects of the question.

Number, one of the gifts of language, enters daily into our lives because it expresses rank, order, greatness, vastness, extent, size, intensity, activity, strength and significance of quantities. Form enters likewise into our daily lives because it limits and visually defines the magnitudes of our experiences. Therefore number and form serve as symbols to express and define the

world of relativity of which we are a part. Therefore mathematics is justified from the sociological aspect of education.

Mathematics promotes accuracy of thought, logical relationships between cause and effect in various kinds of reasoning processes. It gives a means for the interpretation of phenomena and directs the mind to an appreciation of unobserved conditions within and without the realm of the data of experience. Therefore it promotes judgment, imagination and orderly habits of thinking. In applied work it promotes fidelity, poise, commonsense and tact. Therefore mathematics is justified from the psychological aspect of education.

The cardinal virtue of mathematics lies in the fact that in its vast symbolism it is the essence of truth. It is the one great potentiality which underlies and permeates all matter, force and thought. Its non-metrical phase is a measure of the human recognition which has been given to its essential omnipresence.

The meaning of mathematics is relative and varies from the extreme opinion of the biased non-mathematician to the ecstatic feeling of the genius in pure mathematics. Mathematics may be valued as a mere tool, an art of computation or as a transcendent mirror capable of projecting the infinite into the domain of the commonplace and capable of reflecting and revealing the commonplace so as to expose its subtleties and unsuspected beauty.

The non-mathematician regards mathematics as inert and stupefying in the mere majesty of its magnitudes. The mathematician energizes matter with a living reality, gives philology the promise of the fulfillment of its myths, assures the ideals of literature, art and history, endows biologic and physical sciences with an impetus that lifts them out of a sphere of mere nomenclature, interprets for religion the meaning of ancient lore and ecclesiastic doctrine, vitalizes the statistics of social forces and in every sphere of intellectual activity it enters as law and emerges as harmony and beauty. Thus it may be said of mathematics that it is justified from its many-sided purpose for it does enter all the spiritual possessions of the race.

I believe that some of our mathematicians have become so engrossed with their specialty that they have sacrificed their sympathy with the masses. It is this type of specialist who is

a self-styled authority and who compiles a mathematics syllabus so that the whole range of subjects from arithmetic to calculus shall serve no other purpose than to lend support to each other in their apparently unalterable sequence. The baneful influence of such reports is regrettable because such a committee sets itself up as an infallible model and then it requires untiring effort during the decade following to rectify the diverted stream of progressive thought. In sharp contradistinction to such reports as referred to are the extremely valuable reports of the international committee on the teaching of mathematics which are being published by courtesy of the various governments.

If we were convinced that the comprehensive examination would promote the teaching of mathematics we would still find one very important impediment which would have to be removed. I refer to the traditional resentment which one specialist shows toward another specialist on account of the imaginary encroachments and overlapping of the artificial boundaries of their departmental subjects. The time has arrived when these arbitrary divisions of knowledge must be reconstructed so that they may be interlocking and interdependent. It seems to me that the true function of a principal or director of a school will be restored and he will no longer serve as a highly paid clerk if he will, from time to time, unify the several activities of the classroom and test their contributary strength by applying comprehensive examinations of the most general kind. We need information about our problem and we need it very badly, but we do not want mere criticism because the only kind which is of any value is constructive criticism which indicates breadth of view and is representative of the best thought of the world.

There is one other point which should receive our attention because it has a very important bearing upon the unification of the subject matter as now taught in the schools. We have been so conceited in the past as to believe that mathematics is the only subject which could furnish a suitable backbone, or tree, if you like the simile better, upon which to hang all the other subjects. You will recall that Franklin warned his compatriots that if they did not hang together they would hang separately. The same aphorism applies to the artificial divisions of knowledge and explains the loss of Greek and the threatened loss of Latin in the high schools.

I believe that as mathematics teachers we shall have to concede that any subject in the present division of knowledge may be used as the tree upon which to support the other branches of knowledge. If we take this point of view then the comprehensive examination may be likened to one of the great arteries of travel which links us into a great brotherhood and which distributes equitably the gifts of knowledge which the Great Father has bestowed upon us as a divine heritage. Undoubtedly there will be a readjustment of our departments of knowledge in conformity to the spirit of the age. The tendency in education continues democratic in spirit and the growth of knowledge in the individual is not unlike the development of knowledge historically. Knowledge begins with a small nucleus of facts. With the subsequent acquisition of more facts and their interrelation there arises a differentiation and with further growth we have the beginning of the larger divisions of knowledge. Consequent growth means specialization and further subdivision of departmental knowledge. At this stage an overlapping process brings all knowledge into common relation and the whole mass of knowledge is refined, reassembled and reorganized into new departments and the process repeated indefinitely. In view of these facts the comprehensive examination must be at all times a beacon to our progress. One of the advances which has been made in recent years is the presentation of mathematics in one of the forms known as applied, or practical or vocational mathematics. At the present time there are many high school teachers who are preparing corresponding mathematics texts which it is hoped may prove more adaptable to the needs of the recently established comprehensive high school courses. After all has been said and after all the careful pruning has been done we may hope to have mathematics pure and simple, applied and unadulterated. I am willing to admit that a beginning along these lines antedates the present period of discussion and much credit must be given to such typical organizations as the college entrance boards, but they have much to learn both in the setting of the questions as well as in the more philosophic grading of the answers.

In the setting of a comprehensive examination in mathematics the method may be analytic or synthetic. In the analytic method

we collect a group of sterilized questions from which we select a number with special characteristics. In the synthetic method we construct questions so they may possess special characteristics. Such characteristics much emphasize the qualities which we have stimulated in the student. These characteristics will depend upon the kind of students with whom we are dealing and also upon their teleologic training. In a terminal examination it is inexpedient to require an answer which is unrelated to the student's experience and environment. The examination should be graded according to the mental and not the physical age of the student. I believe that the psychologic study of errors will show that the radical defect of mathematics teaching is due to the artificial forcing of latent powers before their natural period of functioning. In consequence of this unnatural procedure we develop a condition of mathematical weakness or even mathematical sterility. If the type of questions are such as would require the use of a book of reference ordinarily, then most assuredly should we permit the use of a reference book during the examination involving that type of question.

There are various categories of qualities, capacities, actions and contents of the mind which we must set down as desirable considerations in preparing and grading an examination. It is evident that these considerations will compel us to resort to some scientific classifications into groups. The following tentative grouping may assist us in estimating the value of these considerations:

Group (*a*) contains the persistent considerations such as accuracy, facility, memorizing number relations, skill in observation, definiteness in information, insight, imagination, reason and judgment.

Group (*b*) contains immediate considerations such as the ability to marshal facts within the subject or in a subject immediately related thereto, clearness of conception in the grasp of principles and in the comprehensive mastery of the subject.

It is not my purpose to outline a definite mode of procedure for the presentation of an examination because I believe that the form of an examination must be determined by the very nature of the qualities or abilities which we may wish to examine. It may be that a brief oral examination will suffice to form a

proper basis of judgment and again it may require a long range of short periodic tests, culminating in a certificate which should set forth in a specific statement the characteristics and abilities of the student. Such a certificate should be supplemented by either a brief interview or a written examination or both, in a formal or informal manner and attested by a careful inspection of the student's work books, and other school products of industry.

The supreme test of our educational labors and of the student's achievements and status may be summarized in the question: Is the student fitted to do the work which he next undertakes? I have endeavored to collect from many sources the qualities which mathematics instruction is supposed to foster and accordingly I have enumerated them below.

(*A*) Command of Resources:

1. Capacity to absorb knowledge.
2. Capacity to reproduce knowledge.
3. Love for knowledge.
4. Self-knowledge.
5. Self-control.
6. Self-respect for virility.
7. Readiness for occasion.
8. Spontaneity.
9. Persistence.
10. Patience.
11. Willingness to submit one's self or one's work to scrutiny.

(*B*) Comprehensive Vision:

1. Breadth of knowledge.
2. Clearness of conception.
3. Ability to co-ordinate.
4. Ability to generalize.
5. Ability to visualize.
6. Breadth of appreciation.
7. Imagination.

(*C*) Critical Acumen:

1. Precise discrimination.
2. Quantitative discrimination.

3. Qualitative discrimination.
4. Judgment.
5. Poise.
6. Initiative.
7. Interpretation.
8. Ability to reason independently.
9. Ability to conduct investigation by analysis.
10. Ability to conduct investigation by synthesis.

(*D*) Constructive Articulation:

1. Ability to marshal facts.
2. Ability to transmit intelligence.
3. Ability to express facts concretely.
4. Ability to originate.
5. Accuracy.
6. Skill.
7. Facility.
8. Mastery of subject.
9. Respect for mental effort.
10. Efficiency in quantitative and qualitative performance.

The list which is herewith appended is intended as a suggestion for a more scientific classification. No attempt is made to define the terms and therefore they are to be interpreted with a flexibility that will make them adjustable to the progressive moods of our civilization. At this point it is well to note that these pretentious claims are subject to the limitations of transferability to other subjects in the light of the criticism of the doctrine of formal discipline.

Before you and I can determine the application of the comprehensive examination upon the student we must subject ourselves to the most rigid comprehensive self-examination on the validity of our claims. In setting a comprehensive examination let us draw upon any of the subject matter which has come within the experience of the student. Then we must set down in some order the topics under the groups (*A*), (*B*), (*C*), (*D*). We should then decide upon the grading of each question and then frame a satisfactory answer which would receive full credit. Then the examination should be put away in cold

storage and the questions and the model answers should be tested and weighed for their merit. If they are satisfactory after this process of inspection try them.

Comprehensive examination for class . . . range of subject matter. . . . Arithmetic, algebra, plane geometry.

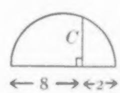
Test 8–*D*7 for speed, time 3 minutes.

Write the answers to the following questions:

1. $\dfrac{\frac{8}{6}}{\frac{2}{3}}$.

2. $\dfrac{3x}{2} = \dfrac{5}{8}$.

3. $c = ?$



The qualities displayed are:

Command of resources, *A* 7.
Comprehensive vision, *B* 1, 6.
Critical acumen, *C* 2, 7.
Constructive articulation, *D* 5, 7.

Test 8–*D*5 for accuracy, time $1\frac{1}{4}$ hrs.

Show the work in full, make extra computations, if necessary, near the right margin and draw a line through such scratch work, indicate the answer and put the work in good form.

1. Temperature Fahrenheit in terms of Centigrade:

F° = temperature in Fahrenheit degrees.
C° = temperature in Centigrade degrees.

$F^\circ = \dfrac{9C^\circ}{5} + 32$; solve for C when $F = 70^\circ$.

2. Construct a line AG three inches in length, divide it into six equal segments marking the points of division in alphabetic order. With D as center construct a circle I with radius $= AD = DG$. With D as center construct a circle II with radius $= BD = DF$. With C as center construct a circle III with radius $= AC = CE$. (*a*) Determine the area included between circles I and II; (*b*) determine the area included be-

tween circles I and III; (c) determine the area common to the circles II and III.

3. Determine the altitude constructed upon the hypotenuse of a 30–60 right triangle when the short leg equals 2.86″. Express the answer with three significant figures.

4. Compute the square of 19.5 by considering it $\left(20 - \frac{1}{2}\right)^2$.

5. Two paper hat boxes have square bottoms. What is the ratio of the areas of their bottoms when their edges are in the ratio of 6:7?

6. Determine the locus of points equidistant from the sides of a parallelogram.

The qualities displayed are:

Command of resources, *A* 2, 3.
Comprehensive vision, *B* 2, 3, 5, 7.
Critical acumen, *C* 2, 3, 4, 6, 7, 9.
Constructive articulation, *D* 3, 5, 6, 8, 10.

Test 8–*A*1 for abstract work, time 1 hr.

1. Solve $x - \sqrt{x^2 - 17} = \dfrac{4}{\sqrt{x^2 - 17}}$

2. Prove: the diagonals of a parallelogram bisect each other.

3. Draw a parallel to the base of a triangle such that its length between the legs of the triangle shall equal the greater segment of either leg.

4. Factor $(x - y - z)^3 - x^3 - y^3 - z^3$.

The qualities displayed are:

Command of resources, *A* 1, 2.
Comprehensive vision, *B* 1, 2, 4, 7.
Critical acumen, *C* 6, 7, 9, 10.
Constructive articulation, *D* 2, 4, 7.

Test 8–*B*3 for co-ordination.

This test should be conducted so that the student has the privilege to consult any book of reference.

1. (*a*) Make a diagram of the room in which you are being examined; (*b*) estimate its length, breadth and height, dimension the diagram; (*c*) from these figures determine the volume of the room; (*d*) make diagrams of the walls which have windows admitting light from the outside, dimension the diagrams estimating the window openings; (*e*) determine the percentage of each wall area, which is used for lighting the room from the exterior; (*f*) estimate the angle at which the light enters them by observing shadows; (*g*) determine the volume of air which must be admitted to the room per hour if each individual requires 25 cu. ft. per min.; (*h*) determine the length of the diagonals of each wall and also of the floor; (*i*) determine the diagonal of the room; (*j*) estimate the weight of the individuals and the furniture upon the floor; (*k*) show how you would estimate the average age of the individuals in the room; (*l*) determine the total age of the individuals in the room from your individual estimate. If this number is used as a date what historic fact would it bring to your mind? (*m*) What kind of work or study do you plan to do next and how is your choice influenced by your knowledge of mathematics.

The qualities displayed are:

Command of resources, *A* 1, 3, 4, 8.

Comprehensive vision, *B* 1, 2, 3, 4, 5, 6, 7.

Critical acumen, *C* 3, 4, 10.

Constructive articulation, *D* 2, 3, 9, 10.

The tests which are outlined above take on a deeper significance according to the maturity of the student but it is evident that a more careful study of the comprehensive examination will show that they may be arranged to reveal a greater range of student character. There should be some supplementary data which represents the student's own reply to typical question blanks ascertained at the time of his entrance to the school. These personality reports should bear upon his experience, previous training and habits. These should be supplemented by an interviewer's report which is a school officer's estimate of the candidate made at the time of his candidacy. Such an interviewer's blank notes the following facts about the candidate at the time of his interview or oral examination: Name; address; date of birth; age; weight; height; appearance (pleasing, neat,

or otherwise); nervous state (quiet, restless or otherwise); manner (attractive, gentlemanly, business-like or otherwise); maturity; intelligence; health; finances; common sense; seriousness; desirability; recommendations of interviewer; date; name of interviewer.

ENTRANCE INFORMATION SHEET.

1. Date ....Course applied for ....
2. Name in full ....
3. Home address ....
4. Height ....Weight .... Age: Years ....Months......
5. Did you graduate from grammar school?..........................
6. How many years did you spend in high school? ...Did you graduate?..
7. When did you leave the high school? ....
8. What other schools have you attended? ....
9. How long in each? ....Did you graduate?..............
10. How many months have you studied algebra?......Geometry?......
11. Physics? ....Chemistry? ....Mechanical drawing?........
12. Where do you expect to live if you enter the Institute? ....
13. Have you ever before taken the entrance examinations at..........
    Institute? ....
14. Occupation of father ....
15. By whom recommended to..........Institute
    Name ....
    Address ....
16. Is the person who recommended you a..........graduate?.........
17. By what company is he employed?..................................
18. By what companies have you been employed? (1).................
    (2) .... (3) ....
19. How many months with each? (1).........(2)........(3)........
20. What was the exact nature of your work with each? (1)...........
    (2) .... (3) ....
21. What were your maximum wages? ....

ENTRANCE INFORMATION SHEET.

No application will be considered unless all information called for is given.

19. Do you have enough money to pay the expenses of the two years?....
20. Have you saved it from your earnings or will it be furnished?......
    ....
21. What kind of work do you prefer?...............................
22. Have you ever had any serious illness?.............What..........
23. Are you in vigorous health now?..................................
24. Are the following normal: sight?......hearing?......smell?......
25. Can you arrange to live within not more than one hour's travelling distance from the Institute?..................................

26. In case not admitted: (1) Can you return to the high school for another year? ....
(2) Can you get a position for a year before again asking admission? ....
27. Why do you wish to take the course selected?......................
28. If it is impossible to admit you to the course you desire, what is your second choice? ....
29. If admitted, do you think you have sufficient earnestness and perseverance to pursue your work uninterruptedly and to devote from three to four hours each evening to the preparation of the work for the following day? ....
....

ENTRANCE INFORMATION SHEET

34. Address ....Birthplace ....
35. Where has most of your life been spent? ....
36. Are both parents living? ....
37. Father's occupation ....
38. To what extent are you interested in machinery, sports, athletics?
....
39. Do you play any musical instrument? ....
40. What books have you read? ....
41. What technical periodicals do you read? ....
42. What study do you prefer? ....
43. What are your religious preferences? ....
44. To what extent do you use tobacco and in what forms? ....
45. Are you strictly temperate? ....
46. On the back of this sheet describe in detail any shop, business, or other practical experience which you have had. State also what tools you can use and give an account of anything you have built.

INTERVIEWER'S SUMMARY.

Name of candidate ....Course ....
Address ....
1. Date of birth ....Age ....Weight .... Height........
2. Appearance: pleasing, neat, or otherwise ....
Manner: attractive, gentlemanly, businesslike or otherwise ....
....
3. Nervous state: quiet, restless or otherwise ....
4. Maturity ....
5. Intelligence ....
6. Health ....
7. Finances ....
8. Common sense ....
9. Seriousness ....
10. Second choice ....

11. Desirability ....
12. Recommendations ....
....
....
....
....
....
....

Signed:

Date:

At the present moment New York City and I dare say many other communities are in the midst of a tremendous educational upheaval and one of the growing vortices is centered about the mathematics of the schools. A most characteristic report has just been issued as a result of the Curtis tests in the New York public schools. Dr. Curtis was engaged by the committee on school inquiry to test the arithmetic abilities of the New York public school children, and he found that, compared with children of schools in other cities, they were slightly better in speed, worse in accuracy and very poor in reasoning. The problems were simple ones in addition, subtraction, multiplication and division. 33,500 students were examined in 903 classes of 52 schools. "So far as any individual child is concerned, to say that he has completed the course in arithmetic in the public schools is to convey no information as to his ability in even the simplest work. He may be almost an absolute incompetent so far as practical work is concerned or he may have acquired a degree of skill that would be adequate for any situation in which he is likely to find himself. . . . It seems probable," says the expert, "that a very simple practical course of study in arithmetic, based directly upon the social needs of the children, would influence for good a greater number of both teachers and children than any other change that could be as easily made." The Norwich tests which were conducted by Professor Terrill, in 1909, showed that the child of today could answer the questions of 1862 with greater satisfaction. My idea is not to test the student by any one form of examination at any one time but rather to institute a series of tests which shall always have a definite predetermined aim in accordance with some of the

elements of our classified educational achievements. A comprehensive examination should be a spectrum of the individual's mind. I believe the time is ripe for the entrance of a great educational reformer and I hope that when he arrives we may know him as none other than an ideal teacher who has made the comprehensive examination a living reality. Our problem can not be answered at this meeting alone because it requires the concerted action of everyone working in the interests of education. It is a great national problem for upon it rests the security and integrity of American citizenship. Let each one of us return to work with a fresh impetus and with a wholesome desire to examine the examinations that we have set in the past and then reset these questions with the larger aims that have been proposed in the discussions of the comprehensive examinations. We will be vindicated by this extra effort and will get a more intimate acquaintance with the student mind. Above all other considerations we will discern that if we draw upon the student's command of resources, comprehensive vision, critical acumen, and constructive ability then we shall be honored by that greatest of appellatives,—a teacher.

HIGH SCHOOL OF COMMERCE,
NEW YORK CITY.

COMPREHENSIVE EXAMINATIONS.

By F. Eugene Seymour.

The present impetus given to the consideration of comprehensive examinations in our schools had its origin in an address by Superintendent Maxwell, of New York City, delivered before the Association of College and Preparatory Schools of the Middle States and Maryland at their 25th annual convention held at Columbia University, New York City, December, 1911. Many liberal interpretations of this particular phase of the address have been made, but I shall define the question and attempt to discuss it, conforming as closely as possible to what seem to be the intentions of the author.

In our present scheme of education in secondary schools the freshman takes his first-year algebra, at the end of the year submits to an examination and, to use the language of the author, promptly "casts the subject into the limbo of despised and forgotten lore." He then takes his second-year algebra, plane geometry, advanced algebra or trigonometry, in the succeeding year, and each in turn after its completion is destined to the same unfortunate neglect. The panacea suggested for this and other consequent evils resulting from such a piecemeal method of examining is a comprehensive examination which shall cover, in this case, all the mathematics he has studied during the four years he has spent in the high school. The burden of this paper is concerned mainly in discussing whether or not such an examination will meet the purposes we have in the teaching of mathematics.

Every properly conducted recitation reflects the aim and purpose we have in teaching. If this aim be the "acquisition of knowledge" it is reasonable to expect this purpose to be revealed in the kind of questions which are asked; if the ultimate aim is to "develop correct habits of thought," again it is reasonable to expect that class work will reflect that aim. It is equally true that every properly constructed examination should reveal the aim and purpose which we have in giving the particular course

over which the examination extends. It is not to be inferred from this that every statement made in the recitation, or every question which occurs upon the examination should bristle with ultimate aim, but, having listened to the recitation, and having carefully read through the examination, it seems a reasonable requirement that we should be able to find reflected in each the aim and purpose we have in teaching that subject.

If we grant this, then the first question to be decided is: What is our aim in teaching mathematics, and secondly is the comprehensive examination the best and surest test as to whether this purpose has been accomplished.

Society is demanding, as never before, and yet in strict keeping with the times, that the students turned out by the schools as finished products shall be capable as citizens to function for their own good and advancement and for that of the community. No more righteous demand could be made, and having accepted this standard it must be our task as teachers of mathematics to add our mite toward this functioning citizen. What can we accomplish through the medium of mathematics? All else being equal, what advantage do we expect that student who has had a training in mathematics will possess over one who has not? What do we understand to be our aim in teaching mathematics?

With the great majority of pupils the training obtained in the elementary and high schools furnishes their sole preparation for the future; to this period we may usually turn and find the reason for the success or failure which attends them in life. Our stewardship here is a profoundly important one, their destiny is literally in our hands. What can we do toward making them efficient citizens?

When we realize that practically none of the majority whose preparation ends with the high school will have any actual need for the mathematics they learn beyond the 8th grade we see at once that for them our aim can not be to "acquire knowledge." It seems folly to attempt to invent problems of every-day life where they would need or could use the facts of algebra and geometry which they have learned. What they have learned in arithmetic, together with sound common sense, will solve the problems they must necessarily meet. The subject matter in algebra and geometry must therefore be treated as a means to

an end and not as the end itself. The facts we learn in geometry are by-products, they are supposed merely to afford us occasions for bringing out the lesson we wish to teach or the power we wish to create. He will not be adjudged a good teacher of geometry who has succeeded in imprinting indelibly upon the memory the facts of geometry, but rather he who has succeeded in creating, through the medium of geometry, a habit of systematic and logical thinking.

There may be inspired by our teaching what is spoken of as the "joy of mathematics," the contact here with absolute truth may have its ethical value, some of the facts may help us to explain and understand phenomena in other sciences, but commendable and helpful as they are these results must be considered as secondary. They all afford us pleasure and satisfaction as we study, but from their very nature they are passing. They serve their purpose in making more agreeable and unconscious the accomplishment of the real task, creating a power which shall endure long after the subject is forgotten.

A carpenter may have an excellent kit of tools and be thoroughly trained in the use of each individual tool, so that if you give him a board and a plane he can reduce it to the smoothness of glass, or if you give him a mortise guage, a mallet and chisel he will be able to make a perfect mortise joint. We would not let this stand, however, as our test of his ability. We would rather insist that he be able to work out any construction we might suggest, say that of a cabinet, and that he show his ability by the proper selection as well as the masterful use of the tools and materials to be employed.

So the student in mathematics may have an excellent kit of tools and be thoroughly trained in the use of each individual tool so that if he be given an exercise under any special theorem or an example under a special type of factoring he will be able to solve both perfectly. We would not let this stand, however, as our test of his ability. We would rather insist that he be able to take a fact disassociated from any fact he has already studied, and systematically select the proper tools he needs to attain the solution.

I realize there are many other things which it is well for us in our teaching of mathematics to seek to attain, but I believe

they are all secondary to the effort to create this power of systematic working and systematic thinking.

It may be argued that the student can not learn to work systematically unless he has tools with which to work and therefore it is essential that he retain his subject matter. If he is to continue his work in mathematics doubtless this should be true; if he is to enter college, as the scheme now is, it has to be true; and yet what advantage does such a student possess over one who has forgotten his subject matter, save that it qualifies him to continue in the same field. If we are going to decide what good it is doing we must not think of mathematics, but turn to life to find it.

I grant it is essential to retain the subject matter during the period when the student is being disciplined into this systematic way of thinking and working, for we must have a field in which to work. But once the habit is created it has served its purpose and its retention can be justified only on the ground that he intends to dig deeper in the same field of learning.

In presenting thus briefly and imperfectly what I consider should be our first aim in teaching mathematics I have been guided by the service it may render him in the future and not by the pleasure he may derive from it in the present. I think our mission is, not to make his mind a storehouse for knowledge, but rather a factory for creating it, with the world, not books, the place to get his raw material.

Granting this to be our aim in teaching mathematics what shall be the surest and fairest test as to whether we have accomplished our purpose?

The advocates of comprehensive examinations argue that under the present scheme of examining the student passes his first year mathematics and promptly forgets the subject matter; he then passes his second year mathematics and promptly forgets it, and so on for the four years. Might we not reasonably add a final stage: he takes his final comprehensive examination and promptly forgets the whole four years? Are we justified in expecting anything else? Why deny him for four years this blessed privilege of forgetting, unless it be that at the end his joy may be complete? We have forced him to retain his subject matter over a period in which he has no use for it save to con-

tinue further in the same field, and then as he stands on the threshold he bids adieu to it all.

Isn't it a blessing to him that he can cast it into the "limbo of forgotten lore?" If there hasn't anything come out of his work in mathematics which has become a part of his very being, which time itself can not destroy, what lasting impression can we expect after the comprehensive examination? And if we have succeeded during the course in inspiring in him correct habits of work which will serve him in the future, what added gain can come from a comprehensive examination? I think it was Lowell who said he was mighty glad he could forget his Latin, so long as he retained the discipline which resulted from its study. The power which comes from the study of mathematics is a growth and can not be created spontaneously by any comprehensive examination. If the power we are seeking to create has not been attained at the completion of each year's work it would be a travesty to seek it by a single stroke.

Again the advocates of comprehensive examinations denounce our present scheme of examination as a persistent foe to thoroughness. They say that our boys and girls having passed an examination never think and never want to think of that particular subject again. If this were true it would be a paradox, for the students make credible progress in their mathematics in their second and succeeding years, and yet this presupposes the knowledge they have acquired during the previous years. Thoroughness doesn't depend upon how much they forget, but upon the quality of the matter they retain. If thoroughness means retention of everything learned in previous years, I am afraid we must all plead guilty to the charge; but if it means retention of essentials, as we know them from experience, and a growing power in systematic reasoning, then most of us are innocent of the charge.

But this isn't all, we are told that "a high school pupil never reviews except under the direction of the teacher, and therefore seldom, if ever, acquires the power and habit of independent review." "This power and habit of independent review," they continue, "is the best preparation for independent power in attacking a new subject, and," they add, "the comprehensive examination that covers all the mathematics in the school is

the only force that will compel independent review and independent study."

I grant that any scheme which would encourage independent review and independent study on the part of the pupil is a most commendable scheme, but I do not believe we are warranted in expecting this to result from the comprehensive examination. The reputation of the teacher as well as that of the pupil would be at stake and I am sure they would resort to all legitimate means to get "prepared" for such an examination. Just as long as there are examinations, whether they cover a period of four weeks or four years, there will be cramming for those examinations. The only difference I can see between the preparation for examinations as they are now conducted and examinations over a period of four years is a difference of degree: the latter would be nothing less than cramming raised to the fourth power.

I heartily believe in encouraging the power of independent review and of independent study, but this is a power which can not be acquired in a day or in a period of six weeks that may be set aside in which to prepare for comprehensive examination. It must come as a part of our daily work and frequent reviews when its purpose is not overshadowed by an impending and all-important examination. It must be a natural and gradual development, not a means to an end but an end in itself.

Let us assume now for the sake of argument that the comprehensive examinations are desirable, and that they are in vogue in our schools. What shall be our scheme of promotion from one grade to the next? Shall it be our present scheme? If so, what importance shall be attached to our comprehensive examinations? In case the pupil fails in his final examination must he become a freshman again and repeat the work or shall he simply beat time until he succeeds in passing another examination?

Each pupil in his senior year will be carrying at least four subjects. According to this new scheme each pupil during his last year will now be required to submit to an examination covering the work of four years in at least four subjects. It would seem for pupils of this age a physical impossibility.

We are at present complaining of the alarmingly small per

cent. of pupils entering school who remain until graduation, and we are trying to devise ways to encourage them to stay and finish their course. I am sure we can all anticipate the inevitable results which would come when the students realize these all-important examinations at the end of the course were hanging over their heads.

Such a scheme, therefore, if adopted, would require, it seems, a readjustment of the whole curriculum and would unquestionably reduce our already too small per cent. of graduates.

Several interpretations have been given to the question of comprehensive examinations, among them that it be a test of ability to use the facts learned in one subject in the study of other subjects. To do this effectively the pupil must work long enough to correlate the knowledge acquired in the different courses. It is questionable if there is enough concentration in our secondary schools to make this of real value. It might be all right for pupils who concentrate their work on certain subjects but this idea of specializing isn't sufficiently marked until they enter college. Another interpretation of the comprehensive examination is that it be a test of ability to use, in a masterful way, the tools they have acquired in any particular subject as manifested in their power of independent and systematic reasoning in that subject. This variety of opinion as to the proper kind of an examination comes from our uncertain purpose in education. The chief aim in an examination should be to find out whether or not we have accomplished our purpose in teaching the subject. When we have correctly decided what this shall be [but not until then], the character of the right kind of an examination will be determined.

Normal School,
Trenton, N. J.

THE COMPREHENSIVE EXAMINATION AS AN ENTRANCE TEST.

By A. H. Wilson.

A recent political essay begins: "There is one great basic fact that underlies all the questions that are discussed on the political platform at this present moment. That singular fact is that nothing is done in this country as it was done twenty years ago."

The volume and intensity of the literature of the last twenty years on the somewhat narrow subject of entrance tests is abundant evidence that the change has been as active in the educational as in the political world. The path of educational advancement in the college is strewn with the corpses of brand new entrance plans, new requirements and new tests. The college catalogue is to be closely scanned year after year for changes in these particulars, suggested by experience or proposed for trial. He is indeed an optimist who, in the light of history, will suppose that the final solution is at hand. In the nature of the case it may be that a final solution is impossible in the educational life of a nation which is itself in the process of development. The problem must nevertheless be attacked when it arises; nor need we doubt that all previous attempts to solve it have been necessary steps, either in contributing a portion of the right method, or in showing what was to be avoided.

In the following discussion it should be said that I have in mind mainly those colleges in the East which do not admit by certificate. The list is not a long one; but I dare say that the number of colleges which regard the certificate system as an experiment is much larger, and all will be interested in any plan which promises a solution of some of the difficult problems of the system of entrance tests.

Allow me to enumerate some of these disadvantages, which have been the theme of discussion of numerous gatherings like this in the last decade.

1. By the large number of entrance examinations and the extensive and minute requirements which have been thought

necessary to safeguard the standard of the college, there has unquestionably arisen a restriction in the freedom of the instruction in the school. I dare say that in mathematics this is a very real difficulty in spite of the recent attempts of both school and college to better conditions. When a teacher is to be judged by the percentage of his class that he passes into the college, he is more than human if he loses sight of the examination before him for any length of time; with the examination as an end in view for both teacher and pupil, the class-room work is likely to partake of the evils of the superficial methods of coaching for a passing mark.

2. In the second place, the examination is not an adequate or a fair test, it is claimed; certainly no one would call it a perfect test. It admits some who have only a superficial knowledge of the subject, and rejects some candidates who should be admitted. It is held at the end of a hard year's work, in a strange place, and on that famous "hot day in June." Its difficulty is a very variable quantity, and may, as is claimed, often run to extremes.

Now some of these drawbacks are inherent in every system of examinations; and a consideration of this fact has caused many colleges to look to the certificate system as a solution of the problem. Nearly all of the reforms hitherto made have had for their object a relief from the difficulties just mentioned. They have nearly all been in the direction of an entire or partial dependence on the certificate system. But with this have entered other serious troubles; an ideal system under the most favorable circumstances, the abuses and difficulties of its administration are only too well known. And anyway, does not a certificate depend largely upon an examination? Held in the school instead of the college, it may lose some of its objectionable features; it has the term grade and the master's knowledge of the boy as a check; but there are certain advantages as well as disadvantages in having the examination set by the college, even in having it in strange surroundings. We shall never be able to dispense with the examination in some form or other; and we greet therefore with enthusiasm a reform which promises the enlargement of the usefulness of the examination, as well as of its fairness as an entrance test.

There are other evils of the present system which I believe

more largely influenced the movement under discussion than those mentioned above.

1. I think that most college men have long deplored the practice of the preliminary examination. Its redeeming feature is the relief it affords the school from the great number of tests, if all these are to be taken at one time. In many cases it may not work harm; but it often does. It often happens, for example, that a freshman presents himself for solid geometry with his plane geometry safely forgotten two years in the past. It has also contributed to the pernicious attitude of the student in regarding the required subjects as so many obstacles to be overcome, to be "done," as the traveller of caricature "does" the foreign cathedral or museum.

2. The practice of cramming for examination with the aid of a skillful tutor, who can in no sense be called a teacher, is universally deplored.

3. A third evil has recently taken a prominent place in numerous confessions of both school and college men; the absence of correlation of studies. Each course is an isolated phenomenon, not a link in a chain, not a part of a coherent whole. We have been repeatedly told by writers who have compared our school system with those of France and Germany, that much is gained for the foreign school in the intimate association of the branches of elementary mathematics. In the same text, one finds both algebra and geometry; plane and solid geometry are frequently studied simultaneously.

4. Finally, as a necessary adjunct to our system, an appalling number of freshmen enter college heavily burdened with conditions, and are thus handicapped in the course at a time when they are most in need of all their energies for the regular work.

What then is the nature of the remedy to be provided for this unfortunate state of affairs? Let us see what can be said in favor of the proposed comprehensive examination.

The idea of the general or comprehensive examination is in the air. We shall hear others today speak of it from a more general point of view, and of its application in a wider field than this. It is by no means a new departure, as we shall learn, but an interesting example of a return to an old method appropriately modified to suit new conditions. So far as I can learn, the dis-

cussion did not originate exclusively in the school, or exclusively in the college; but in the minds of some men in each, who are keenly alive to the problems of modern education, and who have its best interests at heart.

Just what does the term imply? First, in its narrowest application, that the subject matter of the examination shall be not merely the book last used, or the material covered in the last term or in a single course; but the entire subject as studied up to the time of the examination. In a somewhat wider application it would include a number of closely related subjects. An extreme instance is the examination given in our universities for the Doctor's degree, or by the state boards, for license to practice medicine or law. As to quality, the examination must necessarily be different from the ordinary examination; it is not merely a collection of excerpts from the regular examinations; not a mechanical mixture, but rather a chemical compound. The subjects are closely related; the examination must exhibit this fact. The type of question will naturally be more general than in the particular examination; it shall be not so much a test of memory as a test of power, a test of the ability to apply the knowledge acquired and to correlate similar courses.

I do not believe there could be any thought of a correlation of courses in the school beyond the territory of the general subject. In the college, mathematics and mechanics may be combined; or history, economics, and government, as is now done at Harvard; but college men speak of the necessity of special preceptors to direct the correlation process, and of a reorganization of the college courses. It would doubtless be disastrous to attempt any such movement in the school, except in a very limited form; that is a question for the school men to consider. But I do not see why the idea should not be applied to the subject of mathematics taught in the school, and this without serious modification of existing arrangements. Suppose that the subjects are plane and solid geometry, elementary and advanced algebra.

Let us look more closely into the character of the changes, and in doing so I shall take the position of an advocate of the system. Grant first, for the sake of argument, that the examination can be successfully set; just what will be the gain?

I should hope that a sufficient reply would be that the boy

would be better educated, not simply prepared to pass entrance tests. That at any rate corresponds to the effect that the comprehensive examination is expected to bring about in the college. The school men who know the conditions intimately must speak for that; I make the following suggestions subject to correction, but they seem to me to be reasonable. If there is a tendency toward domination of the school course, if the teacher is so rushed for time that he can not vary his regular outlined program, surely it will be a relief to know that, within reasonable limits, a greater emphasis is to be laid on the quality of the preparation; to be assured that the test is not a test of memory chiefly, but of power chiefly. A course proceeding at a 4-theorem speed in geometry is a baneful thing to every true teacher. How often would he not like to assign as a lesson a single interesting original problem (with appropriate hints), outlining its corollaries, its particular cases, its connections with other theorems? There are a host of interesting applications which might be made, models constructed, historical references given; but of these there can now be no thought. I know of one teacher who regularly omits all reference to the classic method of determining π because no examiner in his senses would propose a question like that on a two-hour test. One of the foremost European mathematicians has recently written a book on differential calculus in which the subject is treated entirely by a study of the famous problems which have arisen in the course of its development. The author claims that the student's interest is awakened and nourished by the clothing of the subject in historical flesh and blood, where it would not be engaged by a mere statement of the abstract problem. Who is not familiar with the teacher of the old school whose wide experience and genial wisdom were a liberal education as a by-product of any course. We can not all be that, but we can teach a little more intensively; we can at least let our experience and illustrations include the branches of mathematics which stand adjacent to the one being taught.

Let us look now more specifically at the examination itself, supposing, as I have said, that it has been successfully set.

In the first place the evil of the preliminary is avoided. At the moment of entering college the boy has brought up all his subjects, just as the rare ideal student under the present plan does.

In the second place, the number of examinations is greatly reduced; there will be but one in mathematics. Harvard, which has incorporated this feature in its new entrance plan, requires only four in all.

In the third place, the examination will be the despair of the tutor who relies for his success upon illegitimate methods. The field is too broad, the character of the questions too general, for successful guessing.

In the fourth place, it follows as a corollary that when the examination is passed, entrance mathematics is passed, when the examination is failed, entrance mathematics is failed. There is to be no partial passing, no conditions in mathematics. This principle may be extended to the entire entrance test; either the candidate passes free of conditions, or he fails to pass, and must take all of the examinations over.

How is all this to be brought about? Certainly not without co-operation on both sides. The method contemplates that the most advanced subjects for entrance shall be taught in the last year of the school course. I believe this is the almost universal custom at present; but it requires more. It requires that these subjects shall be so taught that there is a constant automatic review of the elementary subject of the same kind. Now every teacher knows that this can be done, knows how to do it, and no doubt longs to do it; but the iron-clad program capped by the searching examination at the end, that admits of no elasticity. It is just this kind of an examination that it is proposed to eliminate. Let us say that the subject is solid geometry. Numerous simple theorems of plane geometry are used directly in the proofs of the advanced subject. For almost every topic of one there is an analogue in the other. Any fair student who can prove that a sphere can be circumscribed about any tetrahedron, can prove that a circle can be circumscribed about any triangle; we shall suppose of course that his attention has been called to the analogy. A conscious effort to keep the elementary subject fresh in the mind of the student can not fail to produce excellent results. The theorems of most value are those of widest application, as a rule; those that contribute to his further progress. It would be the aim of the comprehensive examination to emphasize in the earlier subject just such theorems.

It may be thought of as an examination in the advanced subject with special designed implication of the elementary. The questions directly on the elementary subject would be of a general nature, involving the principles which receive constant emphasis in the progress of the course. I am trying to make it clear that no great merit is to be attached to the memorizing of proofs; that the examination is to show whether a fair ability to use geometrical and algebraical methods has been acquired. Since it is to be an examination for power rather than for memory, it will be largely composed of applications, problems in algebra and mensuration, and easy originals, with heavy emphasis on the *easy*, in pure geometry.

I have outlined this examination about as I have conceived it. It would in any case be difficult to set; would require considerable time and much thought; but the reward would be great. And I must insist that I think it a practicable scheme. There will be danger of making the examination too easy, of removing its teeth; and there will be a greater danger of making it unreasonably hard. No doubt, at any rate at first, it would be well to include a liberal allowance of optional questions; perhaps, too, to place a somewhat greater weight on the school history of the boy than is done at present.

HAVERFORD COLLEGE,
HAVERFORD, PA.

THE COMPREHENSIVE EXAMINATION IN THE TEACHING OF SECONDARY MATHEMATICS.

By Howard F. Hart.

The problem of this paper is to trace the changes in the subject matter and method of secondary mathematics which would seem to be required by an examination which is to test the student's ability to use his mathematics as tools to accomplish the tasks set him.

It is not a part of this problem to consider whether or not such examinations can be written, to consider whether or not it is possible for a student to cram for such examinaions, to consider whether or not it is justifiable to set examinations in which a half or over of the candidates may fail. Nor is it, furthermore, a part of the problem to consider the specific form of such an examination. Nevertheless it is pertinent for us to observe that all the questions of such a test must be such that their solution does not depend upon the use of tricks or special devices but upon the intelligent use of the regular general methods.

Finally, it is not my purpose to consider any questions which have to do mainly with the related problem of the curriculum.

The general conclusions seem to follow at once from the definition of the terms involved and the limitations of the problem. I believe that the comprehensive examination requires the simplification of the subject matter of mathematics by the elimination of needless repetition, the selection of the best methods of procedure where competing methods exist, and the modernization of certain ancient methods of treatment whose only recommendation seems to be their antiquity. Of more importance is the necessity of the selection of methods of teaching which will develop in the student initiative, independence, and power. Of the highest importance is the change which is necessary in the point of view of the teacher. There seems to be no way to establish these conclusions. We can only make their necessity and meaning clear by concrete illustrations.

One illustration of waste through needless repetition is found

in the more or less complete reproof of previous theorems in a given geometrical proof. This waste is more serious than it would seem to be. It is a common fault in textbooks and can only be obviated by not taking the given proof on faith. The following are three theorems whose common proof is typically faulty in this respect: (1) A line perpendicular to each of two intersecting lines is perpendicular to their plane. (2) If a line intersects a plane, the line in the plane perpendicular to the projection of the first line at the point of intersection, is perpendicular to the line itself. (3) Parallel transverse sections of a pyramidal space are similar polygons whose areas are proportional to the squares of their distances from the vertex.

The present treatment of imaginaries is an illustration of waste from the same point of view. If the imaginary were treated throughout as the product of a real quantity and the unit i the necessity for a separate treatment disappears. It should become a part of a chapter on exponents.

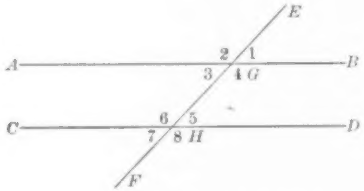
Perhaps the best illustration of waste through the development of competing methods is found in radicals and exponents. The radical sign and its theory are not necessary. I am in favor of dropping them altogether. The time saved would be considerable and would enable us to put more time where it is very much needed, *e. g.*, inequalities.

In considering the modernization of the subject matter one is simply appalled at the prospect. The necessity for it seems to be everywhere. And so I can only hope to call attention to one or two typical examples. First, perhaps, there is that ancient humbug, proportion. When we realize that the writing of it in the common form, which hides its true nature, is the only necessity for most of the terms which we use in it, we should be willing to write it in the form $A/B=C/D$ and to drop that mess of ancient verbiage "antecedent," "consequent," "mean," "extreme," etc., and in time (I almost despair of it) we might find some acceptable substitutes for its remaining ambiguous nomenclature. Proportion is very valuable as a method of reduction of identities and equations but there should not be any theory of proportion as such. The student should know merely, with reasons why, that whenever two fractions are identically equal, certain pairs of derived fractions are also equal.

A close second to proportion is the theory of exponents with its proofs which are not proofs. When we are only seeking to find the *meaning* of certain exponential forms under the principle of *no exception* why should we not frankly say so. It is better mathematics, it is actually intelligible to the average student, and it takes less time.

And then there are the equations. How much longer are we going to apply axioms that do not apply? For a clear concise account of why they do not apply and for a common-sense treatment of the equivalency of equations I refer you to Dr. J. M. Taylor's article in *The Mathematics Teacher* for June, 1910. After reading his paper you will be prepared to enjoy some of our recent texts.

A typical illustration from geometry is the present grouping of the angles formed when two lines are cut by a transversal. Thus if in the figure we consider all angles to be rotated contra-



clockwise and introduce the ideas of initial and terminal arms then the eight angles can be divided into two sets of four each: (1, 3, 5, 7) and (2, 4, 6, 8); since the transversal is a common initial and terminal arm for the set. Usually whatever is true of one angle is true of the set. Thus, *e. g.*, "If two non-vertical angles in one set are equal, then all the angles in each set are equal and the angles of one set are the supplements of the angles in the other," or again, "If two non-adjacent angles in different sets are supplemental the same conclusions follow." By using the ideas of these sets we are able to state the theory of parallels in but four theorems.

I have said that it is of more importance that the methods of teaching should be such as to produce in the highest possible measure initiative, independence, and power. I most emphatically dissent from the idea that the methods of presenting a subject are of minor importance. They are of fundamental importance. The subject matter is mere material. The value of

the result depends upon the methods. For example, I believe absolutely that the reason geometry has been so much attacked is the failure to make it yield initiative, independence, and power by the traditional method. Geometry is ideal material for such purposes and I believe just as firmly that it will yield results when presented by the right methods.

In general, it seems to me that any method must satisfy the following requirements: (1) Anything which is acquired without the use of the reasoning powers has little educational value.* (2) Progress must be through the medium of clear ideas from the known to the related unknown. (3) Initiative, independence, and power can not be taught, they must be acquired through activity. Therefore it seems to me that the methods must be actual rather than philosophic and inductive rather than deductive. By all of which I mean that the student should be given the materials of the subject, just as he is given lumber and metal in the shop, and beginning with their simplest combinations be led to do for himself a list of things selected with regard to the time available, their relation to each other, and their intrinsic value.

In order to show one application of these ideas I have brought along the first lessons of an outline for the beginning of geometry.

Lesson I. Place different kinds of lines (except straight line) on the board and work out the test. Then give the straight line and lead to the definition of line, sect, and point. Straight line axiom, broken line, and curved line.

Home work: Addition and subtraction of sects.

Lesson II. Review: Recall generalizations of previous lesson.

Main topic: Place different polygons on the board and derive classifications (as to number of sides and form). Give usual names.

Home work: Have polygons constructed.

Lesson III. Review: Lessons I and II.

Main topic: Ask class for ideas of an angle. Tabulate and compare. Parts of an angle. Reading an angle, initial and terminal arms.

Home work: Assign intersecting lines and polygons with vertices lettered contra-clockwise, and ask upils to read the angles contra-clockwise.

Lesson IV. Review: Kinds of lines; straight, broken, and curved. Perimeter of polygon as sum of sects.

Main topic: The angles of intersecting lines classified as adjacent,

* Cf. Professor Dewey's "How We Think."

straight, supplementary, vertical, perigon, explementary. Point out how angles are added, infer subtraction.

Home work: Assign polygons with sides produced and have angles (interior, exterior, etc.) read. Draw figures for equal adjacent supplements and explements.

Lesson V. Review: Definitions of classes of angles. Explement and supplement. Give name to right angle of exercise of last lesson.

Main topic: Tests of geometric equality.

(*a*) Senses not reliable in close decisions.

(*b*) Coincidence. Application in "All straight angles are equal."

Home work: A list of axioms of equals. A list of theorems of unequals.

Lesson VI. Review: Lesson V in general.

Main topic: (*c*) Axioms of equals. Application in "A sect has but one midpoint," "An angle has but one bisector," and "All right angles are equal."

Home work: Prove "A sect has but one trisection point from either end" and "All perigons are equal."

Lesson VII. Review: Sum of angles. Complement and perpendicular. Straight line axiom.

Main topic: Coincidence from the point of view of determination.

(*a*) Lines: i. Straight line axiom.
ii. Bisection of an angle.
iii. Perpendicular at a point. Proof inferred as a special case.

(*b*) Point. By two intersecting lines. Proof by the straight line axiom.

Home work: Draw figures for and write out the steps of the previous theorems.

.... etc ....

(This outline was prepared by Mr. William Brubaker, Mr. Elmer F. Conine, and the author, for use in their work in the Montclair high school.) While these lessons are under way no textbook is used. At the close of this work one is furnished in which the material covered here is given in such a way that it can be consulted as a reference. For the remainder of the course practically no readymade solutions are given. The students are led to find their own and having found them, to preserve them in a notebook for use in formal recitations, summaries and reviews, and for future reference.

I have said that a change is required in the teacher. All will agree, I think, that if any change is required it is a change of the highest importance, for the teacher is and must be the chief

factor in secondary education. I do not propose to discuss whether or not a change is required beyond suggesting to any one interested a study of the reports of the various examining bodies, especially the reports of the College Entrance Examination Board, and to repeat a statement made to me within a month by the salesmanager of one of our largest book companies. The statement was to the effect that the book companies found it very difficult to sell any new geometries because the teachers having once become acquainted with the order of propositions and the solutions of the originals of one book would not consider one having a different order or different exercises.

However, if we assume that a change is required then it seems to me that it must lie in our point of view of our relation to the possibility of producing a product able to be tested by a comprehensive examination. I believe that it is being done now and I also believe that it can be done by any teacher who has had sufficient training and uses proper methods. But it must be realized once for all that power can not be taught. It can only come through development gained by self-activity. Under no circumstances can we "tell" into a boy the ability to solve an original geometrical exercise. And to learn and recite the steps of any number of solutions thought out by some one else will not do either. If this seems to be a strong statement we have but to reflect upon the usual horror of students for geometrical originals and their ready ability and interest in algebraic originals. While these exercises are different in kind, and allowance must be made for that, nevertheless the ultimate reason lies in the difference of the ways in which we have attacked the two classes of exercises. As Euclid once said so must we also say that there is "no royal road" to power except through the actual doing of the things over which power is required.

Finally, we must come to appreciate the results of an indifferent stand-pat attitude toward our work. If we are content to go on doing just the same things in the same ways with no thought or question of improvement in time the possibility of change will be beyond us. Let us remember that, "Unto every one that hath shall be given; but from him that hath not, even that which he hath shall be taken away from him."

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WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE COLLEGE?

By C. C. Guthrie.

I was requested to prepare a paper dealing with the question of college preparation in mathematics for pre-medical students. After due consideration, I have decided to occupy the time allotted in sketching some of the conditions encountered by the medical student, and some of the ideals being followed in the leading medical schools, and leave the question of the kind and extent of mathematics to be offered to such students in the college, to the judgment of the college faculty, rather than to take the matter up more dogmatically. For thus it will receive consideration by the men who are best qualified to specify the amount and kind of mathematics to be offered,—not to mention the fact of its being their right and duty. If in presenting the matter, I may seem to get far afield from mathematics, it will be intentional, for only by considering the medical student's needs, and the conditions as a whole, can we meet such special needs most efficiently.

It is generally recognized by students of this subject of pre-medical training, that in addition to the more general college courses, such as English, history, mathematics, etc., the work should comprise at least one year each of physics, chemistry and biology. Also that the student should acquire a reading knowledge of French or German, or preferably both. My own views in the matter are represented in the outline of the combined six-year medical and college course as set forth in page 77 of the general catalogue of the University of Pittsburgh for 1911–1912. This being the case, I believe that the time placed at my disposal can be more profitably occupied by considering in a more general way the conditions to be met and some possible ways of meeting them.

The aim of all such considerations is to make a doctor who is not only equipped to follow the best practices known to medi-

cine in the treatment of the sick, and to look after the welfare of the community as a whole from his knowledge of public hygiene and preventive medicine, but in addition, one who possesses that which we speak of as the result of the so-called cultural training, in order that he may be a good citizen, an integral mainstay of society and a credit to his profession and to his alma mater. It is of course unnecessary to present arguments in justification of these purposes, but I wish to say in respect to the last consideration, namely, in our oft-expressed purpose to train our students so that they may be a credit to our institutions, that we may by the public be considered selfish. But such a conclusion is untenable when it is pointed out that the advancement of our educational institutions is, in the sum total, of far greater benefit to the present and future generations of the community and public at large than to the teaching staffs of such institutions. It is not difficult to convince anyone of the soundness of the obvious arguments which support this assertion.

In order to save time in discussing the relationships that should exist between the high school and preparatory school training and college training in the case of pre-medical students, it will save time if we first briefly consider the nature and scope of the training demanded of such students in the medical school itself.

From the nature of modern medicine, an extensive training in a comparatively large number of subjects is demanded of the student. Only a short time ago it was considered that all subjects of an essential nature could be taught adequately to a student in the short space of two years. Now the time required by all medical schools of standing is four, and in some instances five years. And there is reason for believing that soon a minimum of five years will be the legal requirement of some states. The increase in time required for the medical courses is due to the extension of knowledge in the older branches, and the addition of new ones. And since knowledge in each branch is constantly being augmented, it becomes increasingly more difficult to devote adequate time to each subject in the allotted four years. Thus far, additions to knowledge of any particular subject have, in general, resulted in a demand for additional time in that subject for the student. It is my belief that we are rapidly

approaching the time when all courses will be closely scrutinized with the view of eliminating certain vitally nonessential portions. Indeed this step has already been taken in certain instances. For example, the time allotted to anatomy has been reduced in a number of progressive schools. As the viewpoint of medical teachers broadens out, it will result in a closer correlation of the various subjects and thus automatically any parts of subjects, such as anatomy or physiology, that do not bear an important fundamental, or practical relation to medicine or surgery as a whole will be eliminated as a part of the regular medical course. Thus a certain gain in time may be effected. In the past the greatest time-saving factor has been the putting earlier of such courses as could be given adequately in the college,—such examples are physics, chemistry and biology. To meet this new condition it has been necessary for those schools adopting the plan to require one or more years of college work including courses in such subjects before entering the medical school proper. But even with this arrangement in force the schedule of the medical student is very full. Indeed it is doubtful if the schedule of any school, where the requirement is in force, measures up to the highest pedagogical standards.

Turning now to the question of pre-medical training, we may consider the college.

As above indicated, for those schools in which biology, physics and chemistry are required for entrance, the college must teach the subjects adequately from the medical standpoint. But as it is in the college the student receives what may be termed his academic cultural training, it is undesirable to occupy too much of his time with these basic scientific subjects. To be brief, the work in biology should comprise general botany and zoology. Since, in his medical work, he will devote much time to animal dissection and histology, it is desirable that the major emphasis be laid upon the work in botany. The minimum time for a satisfactory course in biology would be one year.

The course in physics should be of a general nature and include mechanics, heat, electricity and light with especial emphasis on refraction, and sound. The laboratory work should be exacting, and of as practical nature as possible in order that the course may be completed in one year. Chemistry should consist

of general inorganic, organic, and qualitative and quantitative analysis. The time required is two years. The course should be very exacting and like physics be made as practical as possible. In arranging such courses it should be borne in mind by the instructors that the purpose is not to make of the student a biologist or a physicist or a chemist, but that the aim is to give him a sound, adequate and practical training in preparation for his medical studies. If later he desires to specialize in any of these fields, he has the necessary foundational training upon which to build.

Closely related to physics and chemistry is another science that we are now requiring of our combined six-year students, namely physical chemistry. Time does not permit a discussion of the important relation of physical chemistry to medical science, but I may say that many views held in medicine can not be properly understood without a knowledge of the fundamental principles of physical chemistry. And advances in medicine from the standpoint of physical chemistry are constantly being made. The minimum requirement in this subject should be at least one quarter, and in the course considerable time should be devoted to laboratory work.

French and German should be taught at least to the extent of giving the student a practical knowledge of their fundamentals in order that he may read scientific matter printed in these languages, for many important medical works are issued in them. It is true that a number of publications printed in English abstract most of the important articles appearing in French and German, but such abstracts as a rule only give a superficial knowledge of the original papers themselves. In addition, in many communities, it is very desirable that the practising physician be able to speak one or more of the foreign languages.

The importance of training in psychology for the medical student is becoming more and more recognized as its application is leading to improvement in the treatment of certain types of nervous cases. In the college, therefore, he should receive the course comprising the foundational essentials for building up a knowledge in this subject when he comes to the work in psychiatry. Also a short general course in sociology is of great advantage to a student preparing for medicine. Time does not

permit specific mention or discussion of the other courses that the pre-medical student should take in the college.

As in the college, the work of the high school student preparing to enter medicine through the college can be specifically arranged to very great advantage. But I shall only say that it is probably disadvantageous for such a student to take up biology or exact sciences other than mathematics in the high school. For the advantage of such courses to the student who later takes them in the college is at most but slight; and, indeed, he may labor under disadvantage in the college if in the high school he obtained only a smattering or at best a superficial knowledge in the subjects. Also in taking up such subjects in the high school he will expend much time that could, to great advantage, be devoted to such subjects as English, mathematics, geography, history, Latin, etc. In regard to mathematics, whether it is taught in the high school or college, it should be taught as largely as possible as a laboratory subject in order that the student may acquire a concrete and practical knowledge so that when he comes to apply it, as he must do in physics, chemistry and physical chemistry, he may be efficiently equipped. The regular medical student will have no need of mathematical training other than that which is necessary for the work required of him in physics, chemistry and physical chemistry. So in order to plan an ideal course in mathematics for such students, it is necessary to consult the physicist, the chemist and the physical chemist rather than members of the medical faculty.

Of course, students preparing to specialize in certain of the various medical fields, as in the physiology of the special senses, or in the corresponding clinical branches, particularly those dealing with the eye or ear, have need of more extensive preparatory training, especially along the special lines that they propose following. But as the number of such special students is relatively very small, and since such specialization is usually undertaken subsequent to the completion of the regular medical course, and in view of the present overloaded curriculum that the regular student must follow, it seems unwise to require such additional special preparatory courses of the regular student. However, as it is desirable that the pre-medical student be offered the maximum amount of work in cultural subjects, *i. e.*, subjects of

general training and broadening value aside from the courses of necessity fixed in his college curriculum (viz., physics, chemistry, etc., as described above), it may be desirable to include mathematics among such courses—the question being one of the relative value of mathematics in this respect as compared to other available courses offered in the college. The question, therefore, is one for consideration by the college in consultation with a committee from the medical faculty.

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WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE COLLEGE CURRICULUM?

By F. J. Holder.

In the forenoon of this twentieth century, when democracy is asserting itself, no less in the political realm than against the old aristocracy of learning, we who are voluntarily yoked to the common load of teaching mathematics must realize its present state of unrest. Possibly this is not more noticeable in mathematics than in many other branches of the curriculum, and it is probably a bit less conspicuous in the colleges and universities than in the secondary schools; and furthermore, this unsettled condition is by no means confined within the walls of our American institutions, but its constant throbbing is felt in the educational pulse of every progressive country in the world. There seems to be an ever-present desire for a change without first counting the cost of the move; a mere effort to have things different, with no well-defined plan of having them better.

There are a few among us, and many in France, who advocate the substitution of a geometry of motion for Legendre's geometry of congruence, without furnishing sufficient evidence to convince us that the former is not more abstract and ill arranged than the latter. England has recently attempted the elimination of Euclid, with the reward of a temporary mathematical condition that can hardly be satisfactory to any one, and certainly her outfielders have not taken kindly to the play. In many of our private, and not a few state, institutions this disturbance has assumed the form of petty mathematics, ill-arranged courses, or the hope that, except in very special cases, mathematics may be erased altogether from the *required* list of studies prescribed in the college curriculum. Prominent among those entertaining this last fond hope may be mentioned the name of Harvard, which has recently made a forward pass in this direction, and there seems to be considerable argument as to whether or not she has landed safely and is entitled to the lead. Some of us believe there was a fumble involved and prefer to withhold our

applause until after the umpire's decision, which in this case may be given out somewhat later in the game. And so it is here and there and everywhere, we find this longing for something better, but often with the result that something worse appears. I am told that it is not an uncommon occurrence for a discontented child to cry for water when, in reality, the troubled sea needs oil.

I believe it is the duty of the colleges to serve the commonwealth, and many have seen fit to lower, temporarily at least, the general standard in order to meet the needs of the great mass of boys and girls who are coming in from the various high schools and academies, in place of the more selected and, therefore, talented lot which formerly entered. This is sometimes interpreted to mean that we must take off the rich cream which requires sustained effort and logical reasoning and pour out the skimmed milk, subsequently diluted to the proper weakness, and invite the student to drink thereof and grow and wax strong.

Some claim that we should have only such mathematics as is immediately practical, the power and resourcefulness of the potentially practical being lost sight of in the desire for the present need, but I believe that investigation will show that this demand comes largely from technical schools and vocational schools of one kind or another.

Finally there are those who go so far as to assert that mathematics should not appear as a science at all, but that when a real problem arises its solution should be effected and no other problems should be given. Doubtless the adherents of this last group are firm believers in the hand of Providence, and any arguments admissible to a place in this short paper would prove futile and powerless to shake their faith.

As a marked contrast to some of the modern conceptions of mathematics as a science, it might be of interest to give the following quotation ascribed to Novalis (1772–1801) who was *not* a *mathematician* but a German romantic *author,* once of cosmopolitan renown. Carlyle delighted to read and re-read his novels and recommended the author to the world as a clear thinker. Novalis wrote: "The life of the Gods is mathematics. All divine messengers must be mathematicians. Pure mathematics is religion. Mathematicians are the only fortunate ones. The mathematician is naturally an enthusiast. Without enthusiasm, no mathematics."

I freely admit that I do not fully agree with any one of the above-named views for, although I do find a great deal of satisfaction and entertainment in figures at times, I cannot conscientiously bow down and worship the subject as did Novalis. And again, when I observe our modern specimens of model mathematicians who are the authors of our magazine articles and our most familiar text-books, I sometimes find the long hair which is so characteristic of the species, but the long symbolic wings one naturally expects to see upon the shoulders of a divine messenger are not always so much in evidence, and to all outward appearances these Novalis deities have somewhat the semblance of men.

On the other hand, I am not inclined to say that mathematics is intended for its immediate application only; far from it; one must love it for its own sake; but the very fact that it has done so much to shed light upon the laws of nature and to explain the immensity and purpose of the universe and that it is recognized as a power in the removal of the old ideas of idolatry and superstition from among us serves as a mighty shield to protect us from the darts of the skeptic.

The strictly practical men remind us that many of our Carnegies, Morgans, and Rockefellers, though ardent admirers of figures, are not great mathematicians, and they also refer with much pride to prominent discoverors and inventors who have accomplished similar achievements. To such argument, Henri Poincare once replied: "As it is often partially true; bold undertakings are frequently due to those who are free from dizziness, and to prevent dizziness one must not see too clearly. Of these adventurers, only the successful are counted, not those who break their necks." Not the less true is it that modern industrial development, considered as a whole, would have been impossible but for the advancement of science, and to no small degree that of mathematics, which gives form to the dreams of the superficial and renders possible the ideals of the visionary.

In the college curriculum at the present time, we might not be able to require more than one year's work in mathematics, even if we were inclined to try, and the average freshman, in the three, four, or possibly five, hours per week allotted to this subject, can not hope to do more than master the fundamental principles

of algebra, geometry, and trigonometry, so long as he comes to us with the limited preparation now prescribed for entrance to college. It might be well to mention just here that Professor Höfler, of the University of Vienna, advocates the introduction, into the secondary schools, of courses of mathematical instruction which have to do with modern questions of values, for example, the question of the function concept, of "functionally thinking" in mathematics. The late Professor Tannery first raised this question for such schools in France some time ago; then Professor Klein took it up and gave it a definite standing in Germany and Austria, and, while I do not wish to alarm those of us present who are members of our efficient body of high school teachers, I might say that Professor David Eugene Smith, of Columbia University, predicts that we shall be hearing a great deal of this question in the next ten years in the secondary schools of America. I believe the teacher of high school mathematics should be perfectly familiar with such topics as irrational and imaginary numbers, algebraic and transcendent numbers, the relation of mathematics to physics, and the introduction to the calculus. He should have some conception of the significance of number, of space, of intuition in mathematical teaching, and of fundamental concepts, the relation of logic to mathematical proof, and the interesting yet not sufficiently appreciated question of "mathematical sophisms."

In a report submitted on January 8, 1911, to the International Commission on the Teaching of Mathematics, by Committee XII., Subcommittee 1, composed of: Chairman, C. B. Upton, Teachers College, Columbia University; R. D. Bohannan, Indiana University; L. G. Weld, University of Iowa; C. D. Rice, University of Texas; and A. L. Candy, University of Nebraska, it is stated that "The *ideal* preparation for teaching mathematics in the secondary schools and the first two years of college should approach the completeness of the preparation in France and Germany, where one must have a state license, obtained only by passing a most rigid examination, before he can hope to do anything worth while in secondary teaching." In this report, we also find the following prophecy: "As the public is now beginning to recognize that teaching is a profession, a feeling which will undoubtedly increase as the years go by, the time

will undoubtedly come when secondary teaching will be sufficiently attractive financially to enable us to demand from the prospective teacher some such preparation as the following. On the side of pure mathematics, we may expect the calculus, differential equations, solid analytic geometry, projective geometry, theory of equations, theory of functions, theory of curves and surfaces, theory of numbers, and some group theory. On the applied side, we should demand a strong course in mechanics, theoretical and practical astronomy, descriptive geometry, and some mathematical physics with a thorough course in experimental physics. To this should be added special courses on surveying and general applications of mathematics that the student may see to what all of the above work is leading. As pedagogical training, there should be included a strong course on the teaching of secondary mathematics with observation and practice teaching under expert supervision, a course on the history of mathematics, at least one graduate course on the history and teaching of mathematics, and a course of an encyclopedic nature dealing critically with the field of elementary mathematics from the higher standpoint. Such an *ideal* preparation, with the exception of about half of the pedagogic training outlined, is no more excessive and severe than the requirements in France today for the secondary teaching license known as the *agregation,* and we can demand this if the public will give teaching the recognition it deserves.

As I have said before, I believe that the college should serve the commonwealth. By this I do not mean that she should cater to the whims of every individual in the community nor would I intimate that she should attempt to perform all of the educational duties of the city or state in which she is located. It is not my purpose to discredit the work of the technical schools and professional schools of one kind or another, for I am convinced that they have a great mission to fulfill, and many of them are meeting their obligations admirably. It is thought by some that it is the business of the college to furnish executives and teachers for the numerous secondary schools and institutions of higher learning but, while I am proud of the important part she has always played, and will continue to play, in this rôle of honor, I believe she has a broader field of activity in which to

cultivate and harvest her growing products. It is one of her noble functions to make men and not machines. It is true, however, that the college is the foundation or center upon which, or around which, the graduate school and professional schools of the university are built and, while many universities have tried to exist without this factor prominent in their midst, they are coming to realize more and more every year that success or failure, to a greater or less extent, hangs in the balance of this potent factor, and one by one they are installing such a department where it did not already exist.

It is difficult for me to conceive of a college without its department of English and mathematics with their courses (prescribed or elective as the case may be) adorning the curriculum by their presence here and there in each of the four years' work scheduled. When I name certain subjects in pure or applied mathematics with which the ideal teacher of secondary mathematics should be familiar, I believe the college should, in so far as it is possible, provide him with the opportunity to become familiar with such subjects either by offering the individual courses desired or by presenting the ones she does offer in such a way as to give him a thorough knowledge of the fundamental principles underlying the whole scheme and to launch him safely upon the high sea fully equipped with glass and compass and the determination to search out the truth. And when he has found the truth, his preparation should enable him to recognize it, to read it intelligently, to assimilate it, and thus to become master of those things he is supposed to conquer.

It might appear that I am advocating the arrangement of the college curriculum specifically to meet the needs of the secondary school teacher only and am forgetting the large majority of college graduates who do not enter the teaching profession, and immediately the question arises in the minds of those of you who may be intimately connected with well-established teachers' colleges or training schools: "Why don't the colleges refuse admittance to those men and send them to the teachers' colleges where they are fully prepared and equipped to offer this ideal training?" A partial answer to this question is the fact that colleges are more numerous than teachers' training schools, and many there are who find it possible to avail themselves of the

opportunities offered by a near-by college but for whom it would be extremely inconvenient, to say the least, to spend the same time away from home.

All students who seek college training, whether required to take mathematics for one, two, or more years, or left to elect it in the wide open elective system, should not only be given the opportunity of taking such subjects as have been mentioned and others of a similar nature which may appeal to their individual tastes, but they should be brought to realize the "values" in mathematical study. Here the question arises: "What are the 'Values' in mathematical study? What is there in the subject to justify its place in the college curriculum?" I believe the answer should be broad and comprehensive. We should recognize the importance of mathematics as a preparation for the study of other subjects and because of the use we can make of it, that is, for its practical applications; but we should also recognize, as the *underlying reasons* for its study, the seeking of mathematical truths for their own sake, that is, for culture; and for the sake of the training acquired in its study, that is, for mental discipline. By mental discipline, I mean a training in intuition, judgment, memory, imagination, reasoning powers; an improvement in ability to concentrate, to think clearly, accurately, and logically; to recognize the essential elements in a problem, to note relationships, apply principles, and understand cause and effect. I believe that general abilities are gained through the exercise of the mind on a special subject, especially mathematics; in other words, I believe that the power, the sense of mastery, the standards and the ideals acquired through mathematical study are carried over into other subjects and the general activities of life. Were we deprived of the practical aspects of the subject entirely, I should consider the cultural and disciplinary values as necessary and sufficient to justify me in my profession. The modern ideal, however, lies between the two extremes of the strictly logical mathematics of the Greeks and the strictly practical mathematics of the Romans and, while we sometimes insist on the formal study of geometry and algebra, we use all the modern devices possible for presenting the subject and making it clear and vital. A certain standard of logical rigor is not inconsistent with the use of intuition, concrete exer-

NEW BOOKS.

Solid Geometry. Syllabus Method. By E. R. Smith in Consultation with W. H. Metzler. New York: American Book Company. Pp. 403. 75 cents.

As indicated by its title this book was written with the belief that the proofs of geometry should be as far as possible worked out by the pupils either in class discussions or individually. It is however a real geometry for class room use and not a mere syllabus. The order of the propositions is excellent. The arrangement of the exercises (some under the theorems by which they are best solved and the remainder in general groups at the end of sections) is noteworthy. The preliminary section or the approach to the actual theorems is to be especially commended for the following (1) The classification of the theorems of the plane geometry with respect to their use in the solid geometry. This is good because the question of whether the figure must be proved to lie in one plane or not is considered. (2) The discussion of the methods of proof. The best discussion with which I am acquainted. (3) The discussion of the methods of drawing figures. The book is worthy of a trial by those teachers who have been looking for something which will give them the opportunity to get away from the mere memory work of the usual text.

Problems in Educational Readjustment. By David Snedden. Boston: Houghton, Mifflin and Company. Pp. 262. $1.50.

The general problem in this book is how to make education more effective. It implies a consideration of the meaning of culture, social efficiency, liberal education, vocational education, and their relation to general education. The author has singled out particular problems for analysis and discussion and gives a careful treatment of each. The chapter on The New Basis of Method is very suggestive.

Riverside Educational Monographs. Edited by Henry Suzzallo. Boston: Houghton, Mifflin and Company. 35 cents each.

Interest and Effort in Education. By John Dewey.

This is one of the most vital of school questions to-day, as one of the great defects of school life is that the interests and energies of children have not been enlisted in school work as they might be. That interest that fosters development is too little understood and teachers who will read this book will find a very clear statement of its nature, value, and application in school work.

Changing Conceptions of Education. By E. P. Cubberly.

In this volume the author traces the changes in the nature of our life, in the conceptions of the school, and gives some account of the new con-

ceptions in national and educational life with present tendencies. "It is an illuminating historical treatment of the problem of educational reconstruction."

The Ideal Teacher. By George H. Palmer.

Teaching is both a profession and an art, and this volume tells what the qualities of an ideal teacher are, how they are developed, and how used. They are weighty words from a master teacher.

Moral Principles in Education. By John Dewey.

Few have had more influence in reforming school methods than the author of this volume, and any teacher who reads it carefully will get a clearer conception of what the moral principles involved in education are, and a firmer faith in their effective application.

Training for Efficiency. By O. S. Marden. New York: Thomas Y. Crowell Company. Pp. 360. $1.25 net.

This book is packed full of straight-to-the-point talks on how anyone may attain the highest degree of efficiency with the powers at their command. It gives the essence of an inspirational philosophy which is practical and will help anyone to better achievement in their work. Teachers would find their burdens lightened by following its suggestions.

Things that Endure. By J. R. Miller. Edited by John T. Faris. New York: Thomas Y. Crowell Company. Pp. 312. $1.00.

In the words of the author "Nothing that we do for ourselves will endure. There is no immortality for vanity and self-seeking. The glory of self-conceit is a bubble." These are sentiments of a high tone and the book gives much valuable counsel concerning what is worth while in life.

The Glory of the Commonplace. By J. R. Miller. Selected and arranged by John T. Faris. New York: Thomas Y. Crowell Company. Pp. 374. $1.00 net.

A collection of apt and striking illustrations drawn from everyday life and so used that in a few well-chosen sentences a lesson is taught or an inspiration given. The author had a wonderful faculty in this direction, and the book will be found a source of stimulation to better living by those who read it.

The Quest of the Best. By William DeWitt Hyde. New York: Thomas Y. Crowell Company. Pp. 267. $1.00 net.

This book is a joint production by President Hyde and six students working together and treats in a forceful and practical way of the following topics: natural badness the germ of goodness; artificial goodness the repression of badness; the quest of the best; missing the best, sins of excess and defect; the personal motive and the social medium; the

birthright of the child. It is full of suggestion and advice for all who have to deal with boys.

Short Stories of the Hymns. By MARTIN KIEFFER. Lancaster: Steinman and Foltz. Pp. 195. $1.00.

A wider and better knowledge of the circumstances under which our hymns were written would add much interest and make their use more devotional. We have here a brief but splendid account of some forty of the best. It is beautifully gotten up and would make a very suitable gift.

Health and Longevity Through Rational Diet. By ARNOLD LORAND. Philadelphia: F. A. Davis Company. Pp. 416. $2.50 net.

This is a translation from the German edition and gives practical hints in regard to food and the usefulness or harmful effects of the various articles of diet. Dr. Lorand is a man of very large experience, being physician to the baths at Carlsbad, an extensive and observant traveler, and a student of the best scientific authorities, so that he can speak with conviction on these points. Every one should read the book and if they do they will without doubt be benefited by following its suggestions.

Woman in Science. By H. J. MOZANS. New York: D. Appleton and Company. Pp. 452. $2.50 net.

The author first traces the struggle of womankind for things of the mind from the early days of Greece and Rome down to the present time. Then after a chapter on the capacity of woman for scientific pursuits, he treats of her achievements in mathematics, astronomy, physics, chemistry, natural sciences, medicine and surgery, archaeology, and invention. This is followed by a discussion of women as inspirers and collaborators in science and a forecast of her future in the field. There is a comprehensive bibliography and index which give added value to the volume. The author has a fascinating style which makes one slow to lay the book down.

The Blossom Shop. By ISLA MAY MULLINS. Boston: L. C. Page and Company. Pp. 223. $1.00 net.

A story of mother love and sacrifice for a little blind daughter, written in a delightful vein combining humor and pathos. It is a story of the south and its fine sentiment will charm readers of all ages.

The Golden Road. By L. M. MONTGOMERY. Boston: L. C. Page and Company. Pp. 369. $1.25 net.

In this account of the chronicles of a fun-loving group of young people Miss Montgomery has given a very simple and pleasing story though perhaps not quite up to what one hopes for from the author of "Anne of Green Gables." The scenes and people are those of Prince Edward Island.

John O'Parlett's. By Jean Edgerton Hovey. Boston: L. C. Page and Company. Pp. 313. $1.25 net.

This is a novel which appeals to the best in us, which griups our hearts and fills our thoughts. It is a tale of strife and courage where one of the heroes is a dog.

The Marty Twins. By Alice E. Allen. Boston: L. C. Page and Company. Pp. 280. $1.25.

Those who have read the adventures of "Joe the Circus Boy" will want to read this interesting account of the further adventures of himself and his dog, Fritz.

Ralph Somerby at Panama. By Francis Raleigh. Boston: L. C. Page and Company. Pp. 305. $1.50.

Real buccaneers who overran the Spanish main, and adventurers who figured prominently in the sack of Panama, all enter into the life of Ralph Somerby, a young English lad, on his way to the colony in Jamaica. After a year of wandering and adventure he covers the route of the present Panama Canal. The book is not only interesting from the story side, but has a considerable historical value, especially from the notes at the end.

Hawk: The Young Osage. By C. H. Robinson. Boston: L. C. Page and Company. Pp. 272. $1.25.

A fine story of North American Indians. It begins with Hawk a papoose frightened by a bear and follows him until he is finally made chief of his tribe.

Principles of Character-making. By Arthur Holmes. Philadelphia: J. B. Lippincott Company. Pp. 336. $1.25 net.

This is Volume XI of Lippincott's Educational Series and although it is a text on applied psychology and written from a scientific standpoint, parents and teachers in general will find it easy to understand and a book from which they may get much valuable help on this most important work of developing character in children. Its tone is optimistic and its conclusions conservative.

Francis W. Parker School Year Book. Vol. II. The Morning Exercise as a Socializing Influence. Chicago: Francis W. Parker School. Pp. 198. 35 cents.

The Principles of Projective Geometry Applied to the Straight Line and Conic. By J. L. S. Hatton. Cambridge: The University Press, G. P. Putnam's Sons American Representatives. Pp. 366. $3.50 net.

This book is intended to cover all the pure geometry, beyond Euclid, required in order to proceed to an honors degree in mathematics in any

of the leading English universities. The author believes that the student loses much by a neglect of the methods of pure geometry, and an experience of ten years as an examiner leads him to believe that the thorough student of the subject takes a superior place to those who depend upon analysis. It is a welcome addition to this field of geometry and is not only well written but well made.

Modern Electrical Theory. By NORMAN ROBERT CAMPBELL. Second edition. Cambridge: The University Press, G. P. Putnam's Sons American Representatives. Pp. 400. 9/ net.

This edition has been so modified on account of recent knowledge and theories as to be really a new book. It is not intended to be a "popular" work, but is addressed to those students who have a good acquaintance with the older physics and desire to study the more modern developments. The author has "attempted to expound the subject in its logical order, to analyze the arguments by which the various phenomena are correlated, to draw special attention to the assumptions that are made." It should prove an excellent treatment for those who desire a general knowledge of recent developments in electricity.

A Textbook on the Teaching of Arithmetic By ALVA WALKER STAMPER. New York: American Book Company. Pp. 284.

Three things were kept in view in the preparation of this book, viz., the setting of arithmetic (its relation to life), its content, and its method; and it is intended to supply the practical needs of the teacher. It contains much that an inexperienced teacher will find helpful. The bibliography is very meager.

The Meaning of Evolution. By SAMUEL CHRISTIAN SCHUMACKER. New York: The Macmillan Company. Pp. 298. $1.50 net.

Among scientifically educated people the theory of evolution in its broad aspects is an accepted fact. Just what it means to the average person in its bearing on the great concerns of mankind is what the author of this volume has attempted to set forth in the simplest and most understandable terms for the benefit of the reader who has no special training in the sciences. It is a splendid presentation of the case and it would seem that there are few people who would not only enjoy but be benefited by reading it.

At some points the author seems to imply that man in his entirety may have developed from apes. A careful study of Genesis, we think, will show three distinct creations: first material things, second animal life, third human life (distinguished from mere animal life), and there seems to be no good reason for disputing this. Evolution may and undoubtedly does take place within each of these realms, but there is no evidence of an evolution crossing the lines between. We can hardly feel that the essential in man was evolved from apes any more than animal life sprang from material things.

Culture, Discipline, and Democracy. By A. DUNCAN YOCUM. Philadelphia: Christopher Sower Company. Pp. 320.

This is a book of unusual value and presents a strong and consistent educational theory. The author takes the ground that any system of education must exist for the community and state rather than the individual; and that the welfare of the many rather than individual tastes are to be the governing factors in a school curriculum. He states the aim of education to be useful self-activity and that instruction should be most exacting in "the memorizing and retention of the relationships essential not only to direct furtherance of the aim, but to all likelihood of usefulness for general knowledge and discipline.

One rather striking result arrived at is that the usual required subjects in a college course should be made elective and many of the elective subjects required. A subject to be valuable for discipline and culture must have many-sided contact with everyday life, but one of the mistakes the author makes is in assuming that mathematics has little of such contact. The proper study of algebra, the one which is supposed to have the least contact of all mathematical subjects, brings out very clearly the functional relationship which has its application in every phase of human life. It seems rather unfortunate that so many authors assume to speak with more authority concerning mathematics than their acquaintance with the subject would justify.

The Marking System in Theory and Practice. By I. E. FINKELSTEIN. Baltimore: Warwick and York. Pp. 92. $1.00.

Every school and college uses a marking system of some sort and should be interested in every investigation that will tend to an improvement and render the student greater justice. This is the latest and altogether the best discussion of the subject we have seen. Many of the discussions have been theoretical but the conclusions of this are based on a careful statistical study of over twenty thousand marks and are convincing. One of its recommendations is that "every high school and college adopt a five division marking system, based upon a distribution which should, in the long run, not deviate appreciably from the following: Excellent, 3 per cent.; superior, 21 per cent.; medium, 45 per cent.; inferior, 19 per cent.; very poor (failure), 12 per cent." Every teacher should read this discussion carefully and make any needed improvement in their marking system.

Tables and Formulas. By W. R. LONGLEY. Boston: Ginn and Company. Pp. 36. 50 cents.

Computing Tables and Mathematical Formulas. By E. H. BARKER. Boston: Ginn and Company. Pp. 93. 75 cents.

Algebra, First Course. By EDITH LONG and W. C. BRENKE. New York: The Century Co. Pp. 283. $1.10 net.

A combination of algebra and geometry in one course. The book illustrates each algebraic rule by the corresponding proposition of geometry and its figure, and so covers fully the correlation between algebra and

geometry, as well as teaching many geometrical facts. The question as to whether the combination is desirable is an open one, but for one who believes in this method, or who wishes to experiment with it, the book seems to be an excellent one.

Advanced Algebra. By Jos. V. Collins. New York: American Book Co. Pp. 352. $1.00.

An algebra containing a review of first year algebra, and the text for intermediate and advanced algebra, including the college entrance topics and some additions ordinarily considered in a first year college course. The review is intended to be a unification of arithmetic, algebra and geometry, but it succeeds mainly in applying some of the algebra in the settings of the other subjects.

The factoring system on page 23 makes good use of the number of terms in the polynomial to be factored, but it could be made even more valuable by some mention of the numbers of squares and product terms that can give a factorable form in each number of terms.

In the advanced algebra the book has the advantage, from the preparatory school standpoint, of being written with such schools in mind, instead of being a college algebra cut down, as is often the case. It seems too bad that the author did not use simpler methods in some places, as in Location of Roots on page 248, Expectations on page 275, where he still keeps a method which is uselessly complicated, and in Partial Fractions.

The book contains some well chosen historical material.

First Course in Algebra. By William B. Fite. Boston: D. C. Heath & Co. Pp. 285.

This book is an attempt to write a first-year algebra about the equation as a center. It states no axioms but makes explicit and implicit assumptions to cover their omission. Some excellent features are: the discussion of negative numbers, the introduction to division of polynomials, and the combination of type products with factoring. There are a few points where teachers may question the wisdom of methods or statements,—as on page 199 where the statement that a certain equation has no root is likely to create a false impression,—but as a whole the book is very carefully written.

A High School Algebra. By J. W. A. Young and Lambert L. Jackson. New York: D. Appleton & Company. Pp. 508.

An algebra for the first and second high school years, following much the same idea as the elementary algebra by the same authors. It is distinguished by its easy transition from arithmetic to algebra and by its evident attempt to avoid the more complicated examples. The review at the beginning of the second year should prove a saving of time for the teacher, while the postponing of the harder parts of the several topics shows good judgment. The chapter on "Geometric Problems for Algebraic Solution" is excellent, but it requires a good knowledge of geometric formulas to make it of much value.

NOTES AND NEWS.

THE annual meeting occurred as announced in the circular and all the papers listed were given. These will be available later in THE TEACHER. The discussion of the papers was the best within the remembrance of the Secretary. The meeting seemed to "get somewhere" with respect to the questions discussed. The Bibliography Committee gave a preliminary report. The following officers were elected: *President,* Mr. E. R. Smith, The Park School, Baltimore, Md.; *Vice-President,* Prof. H. E. Hawkes, Columbia University, New York City; *Secretary,* Mr. H. F. Hart, The Montclair High School, Montclair, N. J.; *Treasurer,* Dr. D. E. Fitch, The De Lancey School, Philadelphia, Pa.; *Council-Members:* Miss Lao. G. Simons, City Normal College, New York; Mr. W. H. Sherk, La Fayette High School, Buffalo, N. Y.; Mr. F. E. Seymour, State Normal School, Trenton, N. J., in place of E. R. Smith.

The Council was empowered to fill any vacancies. Notice of an intention to amend the constitution was given as follows:

"There shall be an editorial committee composed of an editor-in-chief, a circulation manager, and as many other members as the Council shall each year deem necessary. The term of each member shall be three years. Members shall be appointed in rotation in successive years by the Council."

At the close of the meeting by a unanimous motion the Association thanked the speakers for their efforts and the State Department for its hospitality. H. F. HART, *Secretary.*

AN enthusiastic meeting of the Philadelphia Section was held in the Central High School, Wednesday, October 15, 1913, 3:30 P. M., with the following program:

"Circulating Decimals," Professor Safford, University of Pennsylvania.

"The Mathematics of a High School Course in Physics," Mr. Walter L. Phillips, Principal of Lansdowne Schools.

Discussion opened by Dr. George Stradling, Head of the Department of Science, Northeast High School; Mr. John E. Hoyt Instructor in Physics, Drexel Institute.

An enthusiastic meeting of the Rochester Section was held on November first. There was an attendance of thirty-three and four applications for membership in the Association were received. Officers for the year were elected as follows: *Chairman,* William P. Durfee, Hobart College; *Secretary,* Arthur S. Gale, The University of Rochester; *Executive Committee:* Miss Mary M. Wardwell, Central High School, Buffalo; Miss Eunice M. Pierce, Lockport High School; Harry N. Kenyon, East High School, Rochester.

The following papers were read:

"Graphical Methods of Solving Problems of Elementary Algebra," Charles W. Watkeys, The University of Rochester.

"Some Experiences with a Class of Reviewers in Geometry," Miss Sara C. Walsh, Central High School, Buffalo.

"Class Room Presentation of Types of Numbers," Wilbur H. Cramblet, The University of Rochester.

"Should Elementary Algebra be Readjusted?" William Betz, East High School, Rochester.

The New York Section held its autumn meeting, November 12, with a paper by Professor E. L. Thorndike, of Teachers College, on "The Determination of a Scale for Grading Efficiency in Mathematics." The following officers were elected: *Chairman,* Mr. James C. Brown, Horace Mann School; *Secretary-Treasurer,* Miss Jean Robertson, Normal College.

The Earl J. Early Co., of Philadelphia, have in their *Line-o-graph* a drafting tool that combines a graduated rule, 30°, 45° and 60° triangle, lettering-angle, protractor, compass, irregular curve, and is a wonder of compactness. To see is to admire. There are two sizes, 75c. and $1.00.